Rational chiral Segal CFT:

(1.a) For every closed 1-manifold S, a linear category $\mathcal{C}(S)$ isomorphic to $\mathsf{Vec}_{\mathrm{fd}}^{\oplus r}$ for some $r \in \mathbb{N}$. The assignment $S \mapsto \mathcal{C}(S)$ is symmetric monoidal with respect to disjoint union of 1-manifolds, and tensor product of linear categories.

(1.b) For every closed 1-manifold S, a faithful functor $U : \mathcal{C}(S) \to \mathsf{TopVec}$. The assignment $S \mapsto U$ is a symmetric monoidal transformation from $S \mapsto \mathcal{C}(S)$ to the constant 2-functor $S \mapsto \mathsf{TopVec}$.

(2.a) For every complex cobordism Σ, a linear functor $F_\Sigma : \mathcal{C}(\partial_{\mathrm{in}}\Sigma) \to \mathcal{C}(\partial_{\mathrm{out}}\Sigma)$. These functors are compatible with the operations of disjoint union, identity cobordisms, and composition of cobordisms.

(2.b) For every complex cobordism Σ, and every object $\lambda \in \mathcal{C}(\partial_{\mathrm{in}}\Sigma)$, a linear map $Z_\Sigma : U(\lambda) \to U(F_\Sigma(\lambda))$. The maps Z_Σ are compatible with the operations of disjoint union, identity cobordisms, and composition of cobordisms.

A central ext. $0 \to \mathbb{C}^\times{\times}\mathbb{Z} \to \tilde{\mathrm{Ann}}(S) \to \mathrm{Ann}(S) \to 0$ of the semigroup of annuli with ∂ parametrized by S. The extension depends on the central charge $c \in \mathbb{Q}$.

(3.a) For every $\tilde{A} \in \tilde{\mathrm{Ann}}(S)$, a trivialization $T_{\tilde{A}} : F_A \to \mathrm{id}_{\mathcal{C}(S)}$. The $T_{\tilde{A}}$ are compatible with identities and composition, and the central $\mathbb{C}^\times$ acts in a standard way.

(3.b) For every $\lambda \in \mathcal{C}(S)$, the map which sends $\tilde{A}$ to the composite $U(T_{\tilde{A}}) \circ Z_A : U(V) \to U(F_A(V)) \to U(V)$ is continuous on $\tilde{\mathrm{Ann}}(S)$ and holomorphic on its interior.

For $\mathcal{C}$ a $\dagger$-category and Δ an $\mathbb{R}_+$-torsor, we let $\mathcal{C}\langle\Delta\rangle := \mathcal{C} \otimes \mathsf{Hilb}_{\mathrm{fd}}\langle\Delta\rangle$, where $\mathsf{Hilb}\langle\Delta\rangle$ is the $\dagger$-category of "Hilbert spaces" with $\mathbb{C}\langle\Delta\rangle$-valued inner products, and $\mathbb{C}\langle\Delta\rangle := \mathbb{C}\times_{\mathbb{R}_+}\Delta$. All the $\mathcal{C}\langle\Delta\rangle$ have the same underlying category $\mathcal{C}^\natural$.

Unitary rational chiral Segal CFT:

(1.a) For every closed 1-manifold S, a $\dagger$-category $\mathcal{C}(S)$ isomorphic to $\mathsf{Hilb}_{\mathrm{fd}}^{\oplus r}$ for some $r \in \mathbb{N}$. The assignment $S \mapsto \mathcal{C}(S)$ is symmetric monoidal.

(1.b) For every closed 1-manifold S, a faithful $\dagger$-functor $U \colon \mathcal{C}(S) \to \mathsf{Hilb}$. The map $S \mapsto U$ is a symmetric monoidal nat. transformation.

Δ_Σ is a certain $\mathbb{R}_+$-torsor canonically associated to Σ, which depends on $c \in \mathbb{Q}_+$.

(2.a) For every Σ, a $\dagger$-functor $F_\Sigma : \mathcal{C}(\partial_{\mathrm{in}}\Sigma) \to \mathcal{C}(\partial_{\mathrm{out}}\Sigma)\langle\Delta_\Sigma\rangle$. These functors are compatible with disjoint union, identities, and composition.

(2.b) For every Σ and $\lambda \in \mathcal{C}(\partial_{\mathrm{in}}\Sigma)$, a map $Z_\Sigma : U(\lambda) \to U(F_\Sigma(\lambda))$, compatible with disjoint union, identities, and composition.

For $A \in \mathrm{Ann}(S)$, a lift to $\tilde{A} \in \tilde{\mathrm{Ann}}(S)$ induces a trivialization of Δ_A.

(3.a) For every $\tilde{A} \in \tilde{\mathrm{Ann}}(S)$, a trivialization $T_{\tilde{A}} : F_A \to \mathrm{id}_{\mathcal{C}(S)}$ whose components are unitary. Compatibility with identities and composition. $\mathbb{C}^\times$ acts in a standard way.

(3.b) For $\lambda \in \mathcal{C}(S)$, the map which sends $\tilde{A}$ to the composite $U(T_{\tilde{A}}) \circ Z_A : U(V) \to U(V)$ is strong-continuous on $\tilde{\mathrm{Ann}}(S)$, and norm-holomorphic in its interior.

(4.a) Involutive antilinear natural isomorphisms $f \mapsto f^\# :$

$$\mathrm{Hom}_{\mathcal{C}^\natural(\partial_{\mathrm{out}}\Sigma)}\big(F_\Sigma(\lambda), \mu\big) \to \mathrm{Hom}_{\mathcal{C}^\natural(\partial_{\mathrm{in}}\Sigma)}\big(F_{\overline{\Sigma}}(\mu), \lambda\big)$$

compatible with disjoint union, id'ties, and composition.

(4.b) The maps $U(f) \circ Z_\Sigma : U(\lambda) \to U(F_\Sigma(\lambda)) \to U(\mu)$ and $U(f^\#) \circ Z_{\overline{\Sigma}} : U(\mu) \to U(F_{\overline{\Sigma}}(\mu)) \to U(\lambda)$ are adjoints.

(5.a) The counit $(\mathrm{id}_{F_{D^2}(\mathbb{C})})^\# : F_{S^2}(\mathbb{C}) \to \mathbb{C}$ is unitary.

(5.b) The vacuum vector $Z_{D^2}(1) \in F_{D^2}(\mathbb{C})$ has norm one.