- Yang-baxter equation:

$$R_{12}(\lambda)R_{13}(\lambda+\mu)R_{23}(\mu) = R_{23}(\mu)R_{13}(\lambda+\mu)R_{12}(\lambda) \tag{1}$$

Or in components

$$\sum_{j_1,j_2,j_3} R^{k_1k_2}_{j_1j_2}(\lambda)R^{j_1k_3}_{i_1j_3}(\lambda+\mu)R^{j_2j_3}_{i_2i_3}(\mu) = \sum_{j_1,j_2,j_3} R^{j_2j_3}_{i_2i_3}(\mu)R^{j_1k_3}_{i_1j_3}(\lambda+\mu)Rk_1k_{2_{j_1j_2}}(\lambda) \tag{2}$$

- Yang-Baxter equation for the monodromy matrix:

$$R_{12}(\lambda-\mu)T_1(\lambda)T_2(\mu) = T_2(\mu)T_1(\lambda)R_{12}(\lambda-\mu) \tag{3}$$

- Yang-Baxter algebra:

The Yang-Baxter algebra $\mathscr{A}$ consists of a couple (R,T), where $R(\lambda)$ is an $n^2 \times n^2$ invertable matrix and $T_i^j(\lambda)$ $(i, j \in \{1, ..., n\}; \lambda \in \mathbb{C})$ are the generators of $\mathscr{A}$ that act on some Hilbert space.

$\mathscr{A}$ also contains a co-multiplication Δ:

$$\Delta : \mathscr{A} \to \mathscr{A} \otimes \mathscr{A} \tag{4}$$

Obeying the co-associativity relation:

$$(\Delta \otimes \mathbb{1})\Delta = (\mathbb{1} \otimes \Delta)\Delta \tag{5}$$

In our case Δ works as:

$$\Delta : T_i^j(\lambda) \to \sum_k T_i^k(\lambda) \otimes T_k^j(\lambda) \tag{6}$$

The algebra $\mathscr{A}$ has a mulitplication and a co-mulitplication: $\mathscr{A}$ is called a bi-algebra.

- Second quantization

Define the vacuum state as the state will all spin up: $|0>|++...++>$. Then $\Delta^{L-1}(B(\lambda))$ will act as an creation operator (creating a spin down). From now on call this $B(\lambda)$. A general state will be of the following form:

$$|\Psi_M>=\prod_{i=1}^{M} B(\lambda_i)|0> \tag{7}$$

$$Tr(T_i^j(\lambda))|\Psi_M>=(A(\lambda)+D(\lambda))|\Psi_M>=\Lambda_M(\lambda,\{\lambda_i\})|\Psi_M> \tag{8}$$

- **Commutation relations (the important ones)**:

- $B(\lambda)B(\mu) = B(\mu)B(\lambda)$
- $A(\lambda)B(\mu) = f(\mu-\lambda)B(\mu)A(\lambda) + g(\mu-\lambda)B(\lambda)A(\mu)$
- $D(\lambda)B(\mu) = f(\lambda-\mu)B(\mu)D(\lambda) + g(\lambda-\mu)B(\lambda)D(\mu)$

With:

$$f(\lambda) = \frac{a(\lambda)}{b(\lambda)} \quad g(\lambda) = -\frac{c(\lambda)}{b(\lambda)} \tag{9}$$

- **Reminder**:

- $R^{++}_{++} = R^{--}_{--} = a(\lambda)$, $R^{+-}_{+-} = R^{-+}_{-+} = b(\lambda)$, $R^{-+}_{+-} = R^{+-}_{-+} = c(\lambda)$
- The 6-vertex parametrization: $a(\lambda) = sinh(\lambda+\phi)$, $b(\lambda) = sinh(\lambda)$, $c(\lambda) = sinh(\phi)$

From Rob's presentation:

$$\Lambda_M = a^L \prod_{i=1}^{M} L(z_i) + b^L \prod_{i=1}^{M} M(z_i) \tag{10}$$

With:

$$L(z) = \frac{ab + (c^2-b^2)z}{a^2 - abz} \quad M(z) = \frac{a^2 - c^2 - abz}{ab - b^2 z} \tag{11}$$